Cost allocation in collaborative forest transportation

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Abstract

Transportation planning is an important part of the supply chain or wood flow chain in forestry. There are often several forest companies operating in the same region and collaboration between two or more companies is rare. However, there is an increasing interest in collaborative planning as the potential savings are large, often in the range 5-15%. There are several issues to agree on before such collaborative planning can be used in practice. A key question is how savings should be distributed among the participants. In this paper, we investigate a number of possibilities based on economic models including Shapley value, the nucleolus, separable and non-separable costs, shadow prices and volume weights. We also propose a new allocation method, with the aim that the participants relative profits are as equal as possible. A large case study comprising eight forest companies in Sweden, is described and analyzed.

Keyword: Transportation, OR in natural resources, Supply chain management, Logistics, Economics, Group decisions and negotiations, Linear programming

1 Introduction

Transportation corresponds to a large proportion, about one third, of the total raw material cost in the forest industry in Sweden. Transportation is one part of the wood supply chain, see illustration in Figure 1. It begins at harvest areas in the forests. Here, trees are cut into logs (or bucked) and put in small piles. Piles are characterized by assortments which depend e.g. on species, diameter, length and quality. The number of assortments at each harvest area are in the range 5-15. The piles are collected by forwarders and put in larger piles close to forest roads which are accessible by logging trucks. Transportation can be done in one or two steps depending on whether or not the logs are taken to intermediate storage at terminals before being moved to the customer. When the roads are in good condition, it is possible to make the transportation directly to to the industry. Then, sawlogs are taken to sawmills or ports for direct export, pulplogs to pulp- and/or paper mills and wood fuel to heating plants. The latter is done by special vehicles as it generally requires that the wood fuel is chipped. The main transportation, however, is done by logging trucks for saw- and pulplogs. The principal difference between sawlogs and pulplogs is that sawlogs have a larger diameter. In some cases, trains and ships are involved to support the transportation to and from terminals.

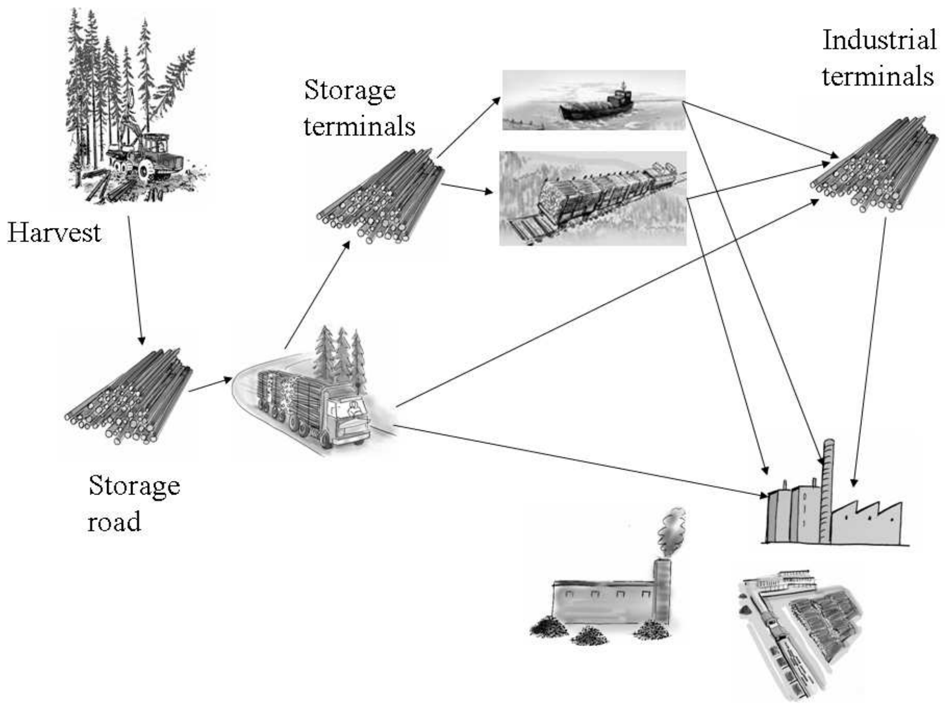

Figure 1: An illustration of the wood supply chain.

The main actors involved in the woodflow are industrial forest-enterprizes, with large forest assets as well as their own pulp and paper industries and sawmills. These can be either private or state owned. There are also forest owners' associations, which represent the private entities and have their own pulp- and sawmills. There are independent sawmills, without any larger forest assets and independent forest owners not connected to any industry. In addition to these primary actors representing the wood producers and wood consumers, there are also the loggers and the transporters, harvesting and carrying the wood from forest to mill. Management can be centralized or decentralized. Even though all actors involved recognize the importance of co-operation and integration along the woodflow chain, it is easy to observe and explain why the different actors, upon optimizing their individual short-term goals, take decisions that that can hinder integration and co-operation. One problem often encountered when trying to co-operate is that planning and decision-making becomes much more complicated.

Large volumes and relatively long transport distances, together with increasing fuel prices and environmental concern makes it important to improve the transportation planning, see e.g. Epstein *et al.* (1999) and Weintraub *et al.* (1996). In many cases, volumes of the same assortment are transported in opposite directions due to a low level of interaction between the forest companies. Supply and demand are generally evenly dispersed geographically for each company. Moreover, as many companies operate in the same region, there is often a high potential for coordination of the wood flow. Coordination creates opportunities to better utilize transport capacity. This can be done by wood bartering and/or backhauling. Wood bartering can be used in such a way that destinations between supply and demand nodes are changed. Backhauling can be used to find better routes by combining two or several destinations (combination of a supply and demand point). In this way, the unloaded distance can be decreased. Examples when wood bartering and backhauling have improved transportation efficiency are found in Forsberg et al. (2005).

Wood bartering (or timber exchange) between forest companies in order to reduce transport cost is fairly common in Sweden. The largest proportion consists of pulp wood exchange since this assortment is normally cut into common lengths, for example 3 meters, and its qualities are fairly equal, no matter where geographically the wood is harvested. Hence, this assortment is easy to exchange. Sawlogs are more rarely used for exchange since the logs are cut in specific lengths depending on which saw mill that will receive the logs. If sawlogs are to be exchanged, the planning for exchange has to be done before the logs are harvested and cut into specific lengths. Timber exchange seldom includes more than two forest companies since the more companies that are involved the more complex the planning becomes. Only volumes are exchanged, and there are no monetary transactions.

In the case when coordination should be included in the planning, certain questions arise: How should the potential coordination be computed? How should the saving be divided among the participants? The first question can be approached by using a Decision Support System (DSS) based on Operations Research (OR) models and methods. The models and methods used in the system FlowOpt (Forsberg *et al.*, 2005) can be used to find the actual saving if all the participants co-operate, as compared to no coordination. The second question is often not addressed. The reason for this is that in practice wood bartering is often only used between two companies and then only with a fixed volume. This means that each company uses its internal DSS to compute the saving for different volume levels without revealing the result to the other company. This can be in order to keep sensitive information away from competitor. In addition, there may be substantial differences in savings among the companies. However, to fully use the potential for coordination the question of division of savings is important.

In this paper, we will address the second question. We will study and use a *cost allocation method* as a tool for allocating costs. We do not split savings, instead we split the common cost among the participants. Further, we do not aim to identify the "best" cost allocation. Instead we analyze a number of alternatives. The most simple solution to a cost allocation problem would be to split the common cost equally, weighted with each participant's volume. However, as will be shown in the experiments, this gives a large difference in relative savings among the participants. We will study a number of different cost allocation methods that are, partly, based on *solution concepts* from co-operative game theory. These methods used are based on the Shapley value, the nucleolus, separable and non-separable costs and shadow prices. When choosing among different solution concepts, we seek one that satisfies specific fairness criteria, called *properties*. A number of different properties are presented in the literature, and an extensive list can be found in Tijs and Driessen (1986). In this context, we will only discuss a limited number of properties.

In a case study carried out in southern Sweden, eight forest companies analyzed the potential savings of integrated or coordinated transportation planning. The potential saving was as high as 14.2%. The purpose of this paper is to develop a framework for participating companies to find values in how the costs (or savings) should be distributed. In our study, we have found that existing approaches do have some disadvantages when it comes to the relative savings. In this paper, we suggest a new approach, called *Equal Profit Method* (EPM). Our motivation was to get an allocation that provides an as equal relative profit as possible among the participants. We also propose a two stage process where we in phase I identify which volumes that do contribute to the savings, and then in phase II we apply the EPM approach.

The outline of this paper is as follows. In Section 2 we describe Linear Programming (LP) based OR models used in transportation planning. In Section 3 we describe a number of economic models used for cost allocation, including EPM. Included is also a small test example to illustrate the characteristics of EPM. In Section 4, we describe the DSS used, the case study and the numerical results. If the proposed methodology is to be used in practice a number of aspects should be considered, and these are discussed in Section 5. Finally, we make some concluding remarks.

2 Transportation planning

Transportation planning in forestry is done in several steps and is divided into strategic, tactical and operational planning. Decisions on a strategic level are influenced by harvesting and road building/maintenance considerations. Tactical decisions mainly concern planning from one week to one year. On an annual basis, transportation is often integrated with harvesting planning, and then often to decide preliminary catchment areas for combinations of industries and assortments. Further, the result of the annual planning is used as a basis to distribute areas to own or sub-contracted transport organizations/ hauliers. A planning problem which often ranges from one to several weeks is to decide the destination of logs, that is, which supply point should deliver to which demand point. The result is used to define transport orders for transporters. In this paper, we focus on this planning problem. The operational planning is to decide actual routes for individual trucks and this is not considered in this paper.

2.1 Linear Programming model

A planning tool often used for tactical planning is the LP model with variables w_{ij} representing the flow from supply point i (with supply volume s_i) to demand point j (with demand volume d_j). A supply point is defined by a location and assortment and a demand point by a location and assortment group. An assortment group is a subset of assortments which state that several assortments may be used to satisfy the demand. The objective is to minimize the total cost where the unit cost of flow between i and j is given by e_{ij} and depends essentially on the distance travelled and has a concave non-decreasing characteristic. We use sets I and J to represent the supply and demand points respectively. The LP model is given as

$$
\begin{array}{llrcll}
[P1] & \min \; z = & \sum_{i \in I} \sum_{j \in J_i} e_{ij} w_{ij} & & & \\
& \text{s.t.} & \sum_{j \in J_i} w_{ij} & \leq & s_i, & \forall\, i \in I \qquad (1) \\
& & \sum_{i \in I_j} w_{ij} & = & d_j, & \forall\, j \in J \qquad (2) \\
& & w_{ij} & \geq & 0, & \forall\, i \in I, j \in J_i
\end{array}
$$

Constraint set (1) gives the available supply for each supply point and set (2) the corresponding demand at a demand point. As not all combinations of supply and demand points are possible due to the definition of assortment groups, we can define the set J_i as the demand points that supply point i can deliver to. In the same way, we define set I_j as the supply points that can deliver to demand point j.

In the LP model above, the cost is based on the fact that the truck drives full from supply to demand point and empty in the other direction. This is the base of standard agreements. However, this gives an efficiency of just 50%. Efficiency would be improved if routes involving several loaded trips were used, i.e. backhauling. Backhauling refers to when a truck that has carried one load between two points, carries another load on its return. The geographical distribution of mills is important in this context. To use backhaulage tours can dramatically decrease the cost, savings between 2-20% are reported in different case studies, see Carlsson and Rönnqvist (2006) and Forsberg (2003). The modified LP model can be written as:

$$
\begin{array}{llrcll}
[P2] & \min \; z = & \sum_{k \in K} c_k x_k & & & \\
& \text{s.t.} & \sum_{k \in K} a_{ik} x_k & \leq & s_i, & \forall\, i \in I \quad (3) \\
& & \sum_{k \in K} d_{jk} x_k & = & d_j, & \forall\, j \in J \quad (4) \\
& & x_k & \geq & 0, & \forall k
\end{array}
$$

This model is column based and the variable x_k denotes the flow in backhaul route k and c_k the corresponding unit cost. The set K represents all routes including direct flows in model [P1] and backhauling. Constraint sets (3) and (4) represent the supply and demand respectively. The coefficients a_{ik} have value 1 if route k picks up at supply point i and 0 otherwise. In the same way d_{jk} has value 1 if route k delivers at demand point j and 0 otherwise. We note that the direct flow variables are represented using columns including only 2 nonzero elements (i.e. one pick up and one delivery). There is often a large number of potential backhaul routes and they can typically not be used explicitly in a solver. Instead, they are generated in a column generation approach (Carlsson and Rönnqvist, 2006). The restrictions imposed on the backhaul routes are e.g. driving time or the number of pick ups and deliveries. In tactical planning, there is often a practical limit for using backhaul routes consisting of two direct flows.

2.2 Coordination between companies

In our case we consider the problem of co-ordinating planning for several companies. It is common that transport costs can be decreased if companies use wood bartering. However, this is difficult as planners do not want to reveal supply, demand and cost information to competitors. In practice, this is solved by deciding on wood bartering of specific volumes. Today, this is done in an ad-hoc manner and is mostly dependent on personal relations. In Figure 2 we illustrate the potential benefits of wood bartering when two companies are involved. Here we have four mills at two companies (two mills each) together with a set of supply points for each company. On the left hand side, each company operates by itself. The catchment areas are relatively large as compared to the right side where all supply and demand point are used on equal terms.

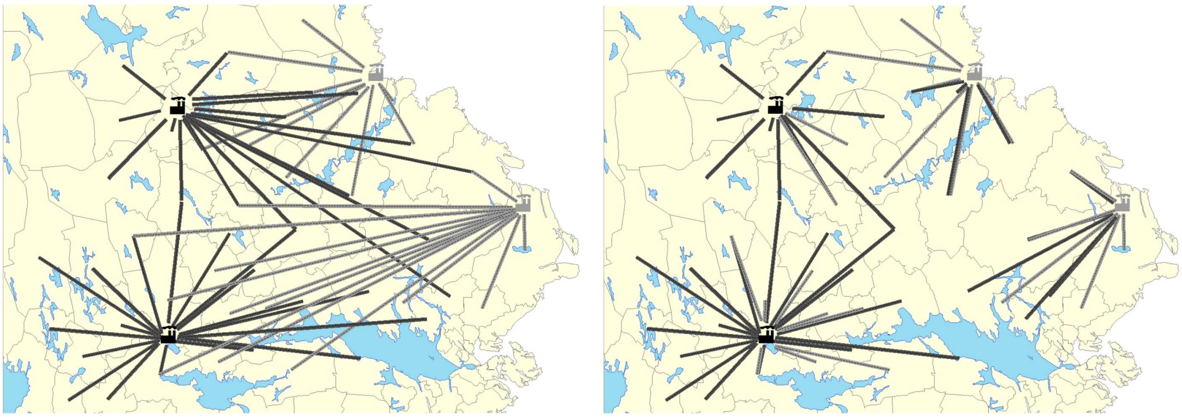

Figure 2: Illustration of wood bartering. In the left part each company operates by itself and in the right part both companies use all supply points as a common resource.

3 Economic models

With quantitative models we can coordinate transportation planning. With more than two companies it is difficult to find bartering volumes of different assortments (between all pairs of companies) and there is a need to find a more quantitative way for how the savings or the total cost should be distributed among the participants. In this section, we describe a number of economic concepts or models that have been used to distribute costs in various industrial areas.

3.1 Basic definitions and Properties

Each solution concept that can provide us with a cost allocation is said to satisfy a number of properties, i.e. fairness criteria. There is, however, no concept that satisfies all criteria listed in the literature. Below we list some of the most commonly used properties. We denote by a *coalition* S a subset of participants, and by the *grand coalition* N all participants. It is assumed that all participants have the opportunity to form and cooperate in coalitions. When coalition S co-operates, the total (or common) cost $c(S)$ is generated. In terms of co-operative game theory this cost function is called the characteristic cost function and each participant is called a player. We say that the cost allocation problem is formulated as a co-operative game.

A cost allocation method that splits the total cost, $c(N)$, among the participants $j \in N$ is said to be *efficient*, that is $\sum_{j \in N} y_j = c(N)$, where y_j is the cost allocated to participant j. A cost allocation is said to be *individual rational* if no participant pays more than its "stand alone cost", which is the participant's own cost, when no coalitions are formed. Mathematically, this property is expressed as $y_j \leq c(\{j\})$.

The *core* of the game is defined as those cost allocations, y, that satisfy the conditions

$$
\begin{aligned}
\sum_{j \in S} y_j &\leq c(S), \quad S \subset N \\
\sum_{j \in N} y_j &= c(N) \quad (\text{ efficiency})
\end{aligned}
$$

That is, no single participant or coalition of participants should together be allocated a cost that is higher than if the individual or coalition acted alone. A cost allocation in the core is said to be *stable*.

For each coalition, S, and a given cost allocation, y, we can compute the *excess* $e(S, y) = c(S) - \sum_{j \in S} y_j$, which expresses the difference between the total cost of a coalition and the sum of the costs allocated to its members. For a given cost allocation, the vector of all excesses can be thought of as a measure of how far the cost allocation is from the core. If a cost allocation is not in the core, at least one excess is negative.

The cost function (or the game) is said to be *monotone* if $c(S) \leq c(T), \ S \subset T \subset N$. Note, that this means that if one new company is included in a coalition, the cost never decreases. The game is said to be *proper* if $c(S) + c(T) \geq c(S \cup T), \ S \cap T = \emptyset$. That is, the cost function is sub-additive. In such a game, it is always profitable (or at least not unprofitable) to form larger coalitions.

3.2 The Shapley value

The Shapley value is a solution concept that provides us with a unique solution to the cost allocation problem. The computation formula stated below expresses the cost to be allocated to participant j, and is based on the assumption that the grand coalition is formed by entering the participants into this coalition one at a time. As each participant enters the coalition, he is allocated the marginal cost, and this means that his entry increases the total cost of the coalition he enters. The amount a participant receives by this scheme depends on the order in which the participants are entered. The Shapley value is just the average marginal cost of the participants, if the participants are entered in completely random order. The cost allocated to participant j is equal to

$$y_j = \sum_{S \subset N : j \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}[c(S) - c(S - \{j\})],$$

Here $|.|$ denotes the number of participants in the considered coalition. The summation in this formula is the summation over all coalitions S that contain participant j. The quantity, $c(S) - c(S - \{j\})$, is the amount by which the cost of coalition $S - \{j\}$ increases when participant j joins it, here denoted by the marginal cost of participant j with respect to the coalition $S - \{j\}$.

The Shapley value is based on four axioms formulated by Shapley in 1953. These axioms express that a cost allocation computed according to this solution concept satisfies the properties of *efficiency, symmetry, dummy property and additivity*. Symmetry means that if two arbitrary participants, i and j, have the same marginal cost with respect to all coalitions not containing i and j, the costs allocated to these two participants must be equal. The dummy property states that if participant is a dummy, in the sense that he neither helps nor harms any coalition he may join, then his allocated cost should be zero. Finally, additivity expresses that, given three different characteristic cost functions c_1, c_2 and $c_1 + c_2$, for each participant, the allocated cost based on $c_1 + c_2$ must be equal to the sum of the allocated costs based on c_1 and c_2, respectively. For an exact formulation of these axioms we refer to Shapley (1953).

The Shapley value provides us with a cost allocation that is unique. However there is no general guarantee that it is stable, e.g. it does not necessarily satisfy individual rationality. It can be proven that the Shapley value is the only value that fulfills the above four axioms.

3.3 The nucleolus

When computing the nucleolus of a game, we identify a cost allocation that minimizes the worst inequity, such that individual rationality is satisfied. That is, we ask each coalition S how dissatisfied it is with the proposed allocation y and we aim at minimizing the maximum dissatisfaction of any coalition. The dissatisfaction of a cost allocation y for a coalition S is expressed by the excess, which measures the amount by which coalition S falls short of its potential $c(S)$ in the allocation y. The nucleolus is the cost allocation, y, that has the lexicographically greatest associated excess vector. For a more formal definition of this concept, we refer to Schmeidler (1969).

The nucleolus exists and is unique. The nucleolus satisfies both the symmetry axiom and the dummy axiom. If the core is non-empty, the nucleolus is in the core i.e. it represents a stable cost allocation. The *pre-nucleolus* is defined as the nucleolus, but it is not required that individual rationality is satisfied. If the game is monotone, it follows that the pre-nucleolus satisfies individual rationality. If the core is non-empty, the pre-nucleolus is also stable.

3.4 Other cost allocation principles

3.4.1 Allocation based on volumes or on stand alone costs

A straight forward allocation is to distribute the total cost of the grand coalition, $c(N)$, among the participants according to a volume or a cost weighted measure. This is expressed by the formula $y_j = w_j c(N)$, where w_j is equal to participant j's share of the total transported volume, or, alternatively, equal to $c(\{j\})/\sum_{i \in N} c(\{i\})$.

When asked, companies claim that this is the preferred model. It is easy to understand, easy to show and it is easy to compute. However, as we will see, it may provide allocations that are not seen as fair. One reason as why this has not been observed earlier is that in normal wood bartering, the actual costs (or common profits) are not revealed.

3.4.2 Allocations based on separable and non-separable costs

In Tijs and Driessen (1986), cost allocation methods are presented, based on the fact that the total cost to be allocated is divided into two parts: the separable and the non-separable costs. Methods based on this idea first allocate to each participant his separable cost, then distribute the non-separable cost among the participants according to given weights. The separable cost is equal to $m_j = c(N) - c(N - \{j\})$, e.g., the marginal cost of participant j, with respect to the grand coalition. In the literature of this field, this is simply called participant j's marginal cost. The non-separable cost that remains to be distributed is then $g(N) = c(N) - \sum_{j \in N} m_j$. Depending on which weights are chosen, we have different versions of the method; the two most straight forward methods are the *Equal Charge Method*, ECM, which distributes the non-separable cost equally, and the *Alternative Cost Avoided Method*, ACAM, that uses the weights $w_j = c(\{j\}) - m_j$, expressing savings that are made for each participant by joining the grand coalition instead of operating alone.

Tijs and Driessen also describe the *Cost Gap Method*, CGM, where the weights are computed according to $w_j = \min_{S: j \in S} g(S)$, where $g(S) = c(S) - \sum_{j \in S} m_j$. This choice of weights can be explained as follows. The separable cost, m_j is seen as a lower bound for the cost allocated to participant j, when joining the grand coalition. The amount $m_j + w_j$ can be seen as an upper bound for the cost allocated to participant j, since it is what he will pay if all other participants pay their marginal cost, in the best coalition S from the view of j. The use of this method assumes that $g(S) \geq 0, \ \forall S$ and that $\sum_{j \in N} w_j \geq g(N)$. Thus, methods based on separable and non-separable costs allocate the costs according to

$$y_j = m_j + \frac{w_j}{\sum_{i \in N} w_i} g(N).$$

A cost allocation that is computed by the ECM or the ACAM satisfies efficiency and symmetry. When the CGM is used, individual rationality and the dummy property are also fulfilled.

3.4.3 Allocation based on shadow prices

In model [P2] described earlier we get dual or shadow prices for each of the supply and demand constraints. We define u_i and v_j as the shadow prices for the supply and demand constraints respectively. When we solve [P2] for the coalition $S = N$ we get $c(N)$. The optimal dual solution has the property $c(N) = \sum_{i \in I} u_i s_i + \sum_{j \in J} v_j d_j$. The distribution of costs in linear production models, in where [P2] is a special case, has been proposed by Owen (1975) and Granot (1986). They show that the core is non-empty and that a solution can be obtained from the associated LP-problem. The solution is based on market prices which in the LP-model are represented by the shadow prices. Each company's contribution can be found by computing its contribution to the dual objective function value. We assume that company c has contribution s_i^c to supply constraint i and d_j^c to demand constraint j. Then we can compute its contribution as $y_c = \sum_{i \in I} u_i s_i^c + \sum_{j \in J} v_j d_j^c$.

3.5 Equal Profit Method

In this project we found some disadvantages with previous allocation models when it came to the acceptance of the cost allocation among the companies. It was difficult not to show that all companies had a similar relative profit compared to the individual cost. In a negotiation situation, it would be beneficial to have an initial allocation where the relative savings are as similar as possible for all participants. This led us to propose a new cost allocation principle. We therefore suggest a new method which is motivated by finding a stable allocation, such that the maximum difference in pairwise relative savings is minimized. We call this the *Equal Profit Method* (EPM).

The relative savings of participant i is expressed as $\frac{c(\{i\}) - y_i}{c(\{i\})} = 1 - \frac{y_i}{c(\{i\})}$.
By the assumption, that a cost allocation is stable, we have that $c(\{i\}) \geq y_i$. Thus, the difference in relative savings between two participants, i and j, is equal to $\frac{y_i}{c(\{i\})} - \frac{y_j}{c(\{j\})}$.
To find this allocation we need to solve the LP problem

$$
\begin{array}{llcll}
\min & f & & & \\
\text{s.t.} & f & \geq & \frac{y_i}{c(\{i\})} - \frac{y_j}{c(\{j\})}, & \forall (i,j) \\
 & \sum_{j \in S} y_j & \leq & c(S), & S \subset N \\
 & \sum_{j \in N} y_j & = & c(N) &
\end{array}
$$

The first constraint set is to measure the pairwise difference between the profits of the participants. The variable f is used in the objective to minimize the largest difference. The two other constraint sets define all stable allocations. Since the objective is a combination between participants, it is not a weighted nucleolus. In the literature of this field, we have not been able to find an allocation method with a similar objective. Therefore, to our knowledge, this allocation method is new.

In the case when the core is empty, we propose to use the so called epsilon-core. We should note that for our test data, this case never occurred. Using an epsilon-core means that we add a minimum penalized slack in the constraints defining the core. The implication of an empty core is that the grand coalition is not stable, i.e., some companies may break out and start their own coalition. By using an epsilon-core we can keep the grand coalition "stable" and note the existence of a coalition that would have an incentive to break out. Alternatively we can seek the maximal number of players in a game for which the core exists. However, how this subgroup of players should be selected remains to be studied in future research.

An illustrative numerical example

In order to illustrate the difference between a cost allocation computed according to the Equal Profit Method (EPM), and cost allocations based on other well-known concepts, we consider a small example which comprises three participants. The cost of the coalitions S, $c(S)$, are given by $c(\{1\}) = 4,\ c(\{2\}) = 7,\ c(\{1\}) = 5,\ c(\{1,2\}) = 10,\ c(\{1,3\}) = 8,\ c(\{2,3\}) = 10,\ c(\{1,2,3\}) = 12$.

The mathematical model based on the EPM can now be stated as

$$
\begin{array}{llllllll}
\min & f & & & & & & \\
\text{s.t.} & f & \geq & y_1/4 & - & y_2/7 & & (i) \\
 & f & \geq & y_1/4 & - & y_3/5 & & (ii) \\
 & f & \geq & y_2/7 & - & y_1/4 & & (iii) \\
 & f & \geq & y_2/7 & - & y_3/5 & & (iv) \\
 & f & \geq & y_3/5 & - & y_1/4 & & (v) \\
 & f & \geq & y_3/5 & - & y_2/7 & & (vi) \\
 & y_1 & & & & & \leq 4 & (vii) \\
 & & & y_2 & & & \leq 7 & (viii) \\
 & & & & & y_3 & \leq 5 & (ix) \\
 & y_1 & + & y_2 & & & \leq 10 & (x) \\
 & y_1 & & & + & y_3 & \leq 8 & (xii) \\
 & & & y_2 & + & y_3 & \leq 10 & (xii) \\
 & y_1 & + & y_2 & + & y_3 & = 12 & (xiii)
\end{array}
$$

Here y_i is the cost allocated to participant i and f the maximal pairwise difference of relative savings. A unique optimal solution to this model is $y_1 = 3,\ y_2 = 5.25,\ y_3 = 3.75$, resulting in a relative saving equal to 25% for all three participants. Thus, the objective value f is equal to zero. If we compute the cost allocation based on the nucleolus for this small example, we obtain $y_1 = 3,\ y_2 = 5.5$ and $y_3 = 3.5$. Further, the cost allocation based on the Shapley value coincides with the one based on the nucleolus. However, in general, these points are different, as can be observed in the computational results in the next section of this paper.

These cost allocations can be illustrated in a triangle, plotted in barycentric coordinates, in which each point represents a suggested cost allocation (y_1, y_2, y_3), such that $y_1 + y_2 + y_3 = 12$ (see figure 3). The vertices of the triangle are defined by the intersections of the lines corresponding to the constraints (*vii*)-(*ix*), resulting in (0,7,5), (4,3,5) and (4,7,1).

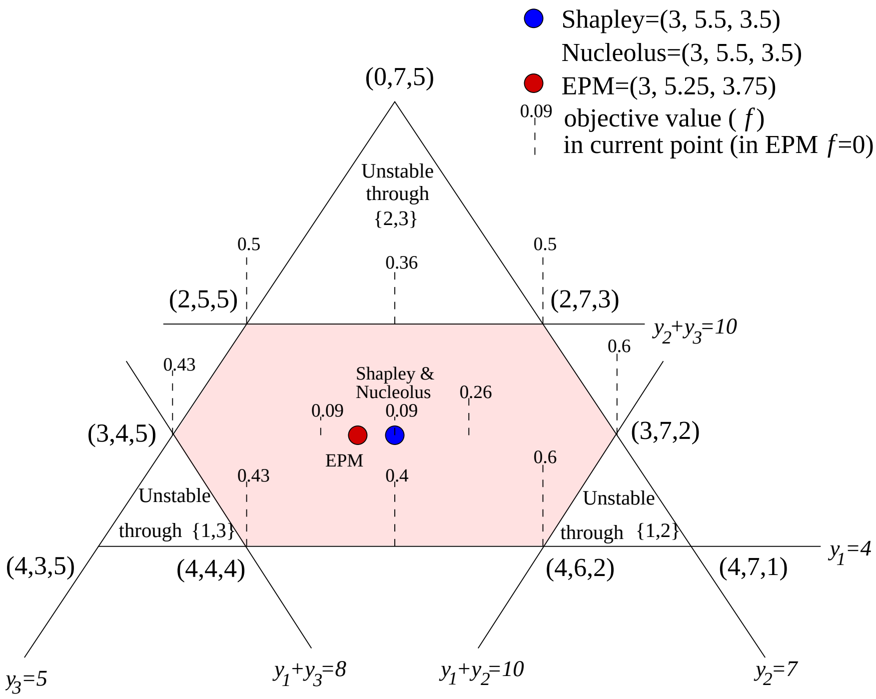

Figure 3: Geographical illustration of the solutions from the different allocations.

In this triangle, the line $y_1 = 4$, for example, is the same as the line $y_2 + y_3 = 8$. The lines corresponding to the constraints (*x*)-(*xii*) are then added to the triangle. The shaded area represents the core (the stable allocations). The points representing the EPM, the Shapley value and the nucleolus are shown. In order to simply illustrate the shape of the objective function value, f, we show this value for a number of points in the core. In the point representing the Shapley value/the nucleolus this value is 0.09, which corresponds to relative savings equal to 25%, 21.4% and 30% for participants 1, 2 and 3, respectively.

Finally, in Table 1 we observe the excess vectors of the two points. Recall that the excess of a coalition is the difference between the total cost of a coalition and the sum of the costs allocated to its members. The two smallest elements of the two excess vectors are equal to 1 for both points. The Shapley value/the nucleolus has a lexicographically greater associated excess vector than the EPM, since the third smallest element is equal to 1.5, which is greater than 1.25.

Coalition:	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}
Shapley/Nucleolus	1	1.5	1.5	1.5	1.5	1
EPM	1	1.75	1.25	1.75	1.25	1

Table 1: The excess vectors of the two solutions.

4 Numerical results

4.1 DSS system FlowOpt

We use the system FlowOpt (Forsberg et al. 2005) developed by the Forestry Research Institute of Sweden (Skogforsk). The system consists of four separate elements illustrated in figure 4.

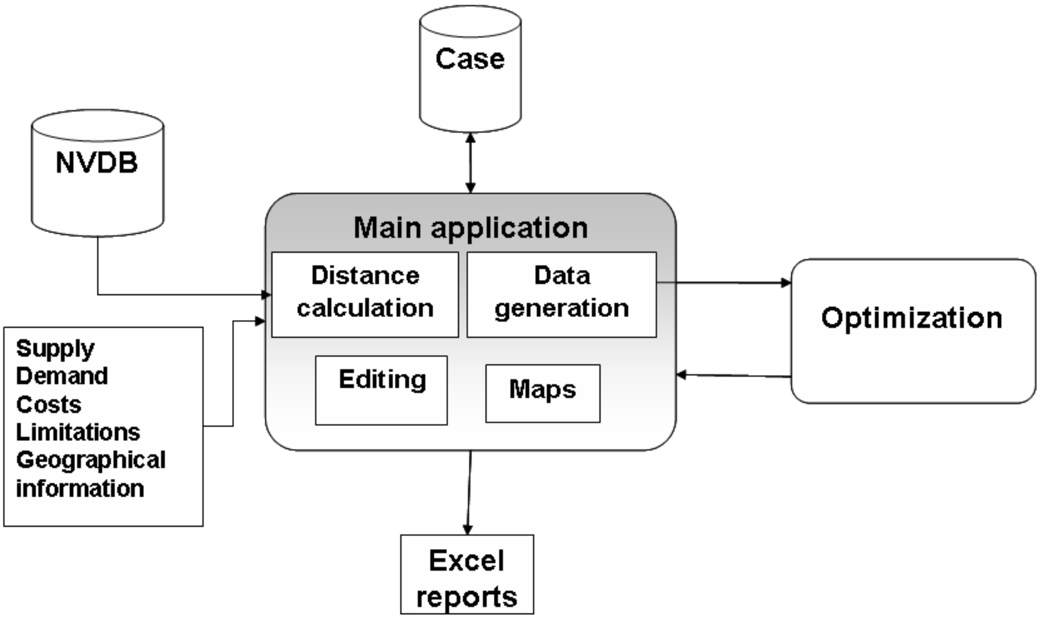

Figure 4: Overview of the FlowOpt system.

The ”Main application” is the central element and is connected to a database storing the data about supply, demand, nodes, transport system etc. The interface offers different functionalities for viewing geographical data and results, report generation and editing the data. Information about supply and demand, for example, is company specific and denoted ”Raw information”. Road information from the National Road database (NVDB) is used when distances are calculated. All information necessary for the analysis is stored in a separate database, denoted ”Case”. The optimization module to solve problem [P2] is located in a separate application. The model in FlowOpt is more general than [P2] as it also can include train transportation and storage over multiple time periods. All data generation for the model are done in ”Data generation”. All data is then translated into a mathematical model by use of a set of input/output routines and the AMPL modeling language (see [2]). As a solver we make use of the CPLEX-optimization system (see [7]). Results from the optimization module are then imported back into the main application where different report options are possible. Normally, the results are exported to Excel or further calculations are done in a database in order to analyze specific key figures. The results are also interpreted in the main application where the viewer chooses different wood flows to show in the GIS system. The viewed wood flows can also be exported as shape files in order to make more advanced presentations in commercial GIS tools.

4.2 Case study

The data used in this paper has been taken from a study done by the Forestry Research Institute of Sweden for eight participating forest companies. The companies operate in southern Sweden and cover different geographical areas. Some companies cover the entire region and others only a part. In figure 5 we provide an example of the area covered by three companies. It is clear that if the company to the left collaborate with the middle company, the cost savings would be small. However, if either of the two or both should work with the company to the right the potential would increase dramatically.

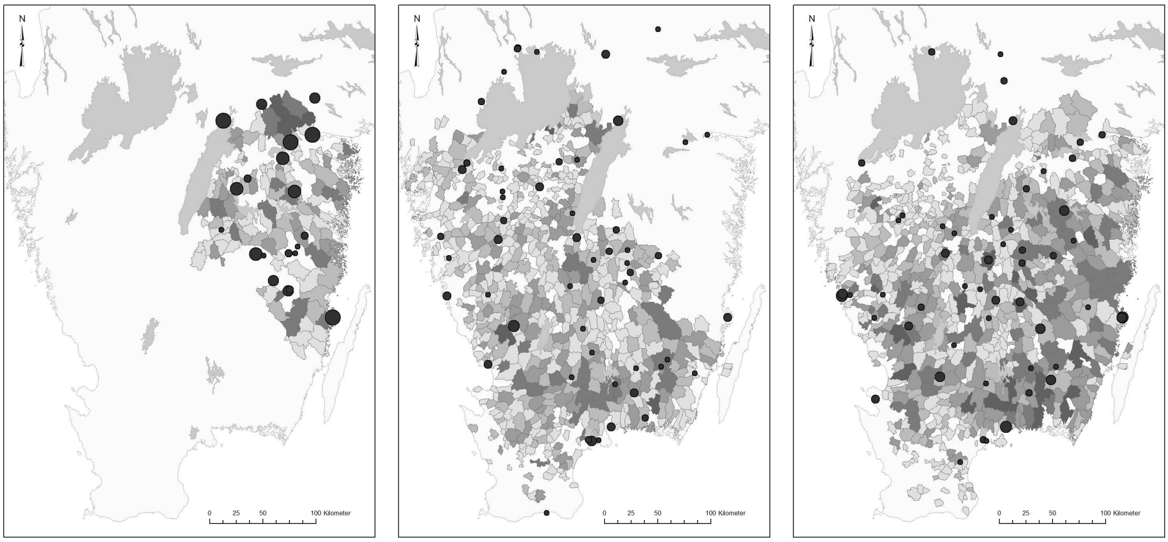

Figure 5: The operating areas for three companies. Darker regions indicate areas of supply points and circles demand points.

The data is taken from transports carried out during one month. It involves all transports from the eight companies and includes information on time, from/to nodes, volume and assortment. There is also a large difference in size between the companies. Table 2 shows the volume transported for each of the companies.

Company	Volume	Proportion (%)	# Supply points	# Demand points
Company 1	77,361	8.76	416	37
Company 2	301,660	34.16	1700	87
Company 3	94,769	10.73	466	17
Company 4	44,509	5.04	308	18
Company 5	232,103	26.29	1468	105
Company 6	89,318	10.12	275	30
Company 7	36,786	4.17	175	9
Company 8	6,446	0.73	34	7
Total	882,952		4842	310

Table 2: Information on the volume (m^3), proportion (in %), number of supply and number of demand points for each company.

In order to make a comparison, the same distance table and cost functions are used for both optimized and for the actual transportation carried out. There are several comparisons that are interesting and we compute the following results.

Real	Single company with actual operated flows.
opt1	Single company with direct flows.
opt2	Single company with backhauling flows.
opt3	Full coordination with direct flows.
opt4	Full coordination with backhauling flows.

With the above results we can compare the potential of coordination by comparing *opt1* and *opt3* (or *opt2* and *opt4*). We can also estimate the overall potential savings by comparing *Real* with *opt4*. To compare the manual solution against the backhauling is difficult as the information about the actual transportation does not include information about backhauling. Therefore we do not know the extent to which backhauling was used in practice.

In Table 3 the results from each of the scenarios is given. We note that the savings obtained from solving [P2] for each individual company and for a full coordination, respectively, provides a saving of 8.3% (*opt1* → *opt3*) or 8.8% (*opt2* → *opt4*). Further, if we consider the potential savings as compared to the actual transportation, we get a saving of 14.2% (*Real* → *opt4*).

Company	*Real*	*opt1*	*opt2*	*opt3*	*opt4*
Company 1	3,894	3,778	3,640		
Company 2	15,757	14,859	14,684		
Company 3	4,828	4,742	4,703		
Company 4	2,103	2,067	2,043		
Company 5	10,704	10,340	10,153		
Company 6	5,084	4,959	4,826		
Company 7	1,934	1,884	1,877		
Company 8	0,333	0,333	0,332		
All companies				39,253	38,315
Total	44,637	42,963	42,257	39,253	38,315

Table 3: Costs (in kSEK) for the scenarios analyzed.

4.3 Test instances

To analyze the cost allocations and their impact, we have constructed eight instances. These are described briefly with the main characteristics in table 4. Instance I1 is the original data for all companies and the entire month. Instances I2-I5 are the weekly volumes transported. Instance I6 is generated by adding the costs of each of the instances I2-I5. Note, that instance I6 coincides with instance I1 in the number of supply points, number of demand points, period and the transported volume, but not in the transportation cost which is computed with FlowOpt (see below). In instance I7 we use filtered data where some supply and demand volumes are removed. We compare the flow of the transported volumes in *opt1* and *opt3*, that is, in the case with no coordination between companies and in the case of full coordination (corresponding to instance I1). The part of the transported volume that coincides in both cases is removed. By this we can analyze each company's real contribution to the collaboration. Finally, in instance I8 we have removed one company from the test data.

Instance	# companies	Period	Volume (m^3)
I1	8	weeks 1-4	882,952
I2	8	week 1	217,532
I3	8	week 2	180,179
I4	8	week 3	210,862
I5	8	week 4	274,379
I6	8	sum of weeks 1-4	882,952
I7	8	weeks 1-4	384,726
I8	7	weeks 1-4	805,591

Table 4: Main characteristics of the instances used in the experiments.

As a basis for all the experiments, FlowOpt has been used to solve the transportation problem [P2] for each possible coalition of companies. The number of different coalitions for eight companies is $2^8 - 1 - 8 = 247$. The total transportation cost obtained for each coalition S, is then used to initiate the value of $c(S)$. In addition we have solved each of the single company problems giving a total of 255 problems solved for each instance.

The optimization models vary in size and the problem representing the full coalition with direct flows in instance I1 has 5,053 constraints. There are 211 demand and 4,842 supply constraints respectively. The reason why the number of demand constraints are 211 and not 310 (which is the overall number of demand points summed over individual companies) is that some companies deliver to the same industry (and assortment group). There are 39 different assortments and 12 assortment groups. The number of direct flow variables is about 240,000. The solution time for this case is a few seconds.

The size of the optimization model increases dramatically and the model representing the full coalition with backhaul flows has as many constraints as before i.e. 5,053 constraints but the number of potential backhaul variables exceeds 100 million. In the column generation we typically generate about 100,000 new variables (beside the 240,000 representing direct flows) before it has converged. The solution time for each such problem is about 3 hours for the full coalition. The overall time to solve the 255 backhaul problems for I1 exceeds 600 computational hours.

Cost allocations for instance I1

In Table 5 we show cost allocation results according to the different concepts described. In the column "Individual" the companies' costs of operating alone are shown. In the remaining columns we give the cost allocations obtained by the concepts and methods. For each computed cost allocation, the savings as compared to the individual costs are given. The average saving is 8.6%. We also provide information as to wether the allocation is stable or not.

The cost allocations computed according to Volume, Shadow and ECM are not stable.

Company	Individual	Volume	%	Shapley	%	Shadow	%	ECM	%
Company 1	3,778	3,439	9.0	3,586	5.1	3,622	4.1	3,808	-0.8
Company 2	14,859	13,411	9.7	13,528	9.0	12,969	12.7	12,890	13.3
Company 3	4,742	4,213	11.2	4,102	13.5	4,069	14.2	3,973	16.2
Company 4	2,067	1,979	4.3	1,889	8.6	1,791	13.3	2,093	-1.3
Company 5	10,340	10,318	0.2	9,747	5.7	10,531	-1.8	9,643	6.7
Company 6	4,959	3,971	19.9	4,503	9.2	4,377	11.7	4,521	8.8
Company 7	1,884	1,635	13.2	1,587	15.8	1,591	15.6	1,735	7.9
Company 8	0,333	0,287	14.0	0,310	6.9	0,303	9.1	0,588	-76.5
Sum	**42,963**	**39,253**		**39,253**		**39,253**		**39,253**	
Stable		**No**		**Yes**		**No**		**No**	

Company	Individual	ACAM	%	Nucleolus	%	CGM	%	EPM	%
Company 1	3,778	3,620	4.2	3,650	3.4	3,623	4.1	3,523	6.7
Company 2	14,859	13,464	9.4	13,207	11.1	13,441	9.5	13,549	8.8
Company 3	4,742	4,090	13.8	4,081	14.0	4,082	13.9	4,324	8.8
Company 4	2,067	1,906	7.8	1,935	6.4	1,909	7.6	1,884	8.8
Company 5	10,340	9,732	5.9	9,848	4.8	9,743	5.8	9,428	8.8
Company 6	4,959	4,511	9.0	4,546	8.3	4,520	8.9	4,522	8.8
Company 7	1,884	1,615	14.3	1,667	11.5	1,620	14.0	1,718	8.8
Company 8	0,333	0,314	5.7	0,318	4.6	0,315	5.6	0,304	8.8
Sum	**42,963**	**39,253**		**39,253**		**39,253**		**39,253**	
Stable		**Yes**		**Yes**		**Yes**		**Yes**	

Table 5: Distribution of costs and their savings as compared to the individual costs for instance I1 using direct flows.

It is interesting to note that according to the cost allocation based on the ECM, a number of companies lose, compared to when they operate alone. The reason for this is that the weights used to allocate the non-separable cost do not take into account each company's individual cost and savings. Also, the difference in savings for each company varies greatly with the concepts Volume (0.2%-19.9%) and ECM (-76.5%-16.2%).

The cost allocation based on shadow prices does not seem to be appropriate, as one of the largest companies does lose. There can be several reasons for this. One is that the transportation problem [P2] has multiple dual solutions and we use only one in the tests.

The Shapley value, ACAM, nucleolus, CGM and EPM are stable. We observe that the core is non-empty and, therefore, at least the nucleolus must represent a stable cost allocation. The fact that the Shapley value, the ACAM, CGM and the EPM also do so is interesting. The cost allocations obtained by the Nucleolus, ACAM and the CGM are similar and have a similar range (about 4%-15%) of the savings. The extra information about coalitions that is added in CGM, obviously, does not have any large effect on how the total cost is divided among the companies.

The most evenly spread cost savings of 6.7%-8.8% are, as expected, produced by the EPM, where the maximal pairwise difference is minimized. The reason why the value is not equal for all is that there is no such stable cost allocation. This behaviour is studied further in instance I7 where company 1 is removed from the coalition.

Effect of backhauling

Next we consider the case when backhauling routes are used. Here, we limit our choice of cost allocation concepts to the Shapley value and the EPM. In table 6, we report on the numerical results obtained for instance I1. The average saving is increased from 8.6% to 10.8%. In the table, we give the savings based on the individual backhauling costs. The results for the two concepts are very similar in structure. Companies 3 and 4 increase the savings more and this is probably due to the geographical distribution where their supply and demand can be included in backhauling to a larger extent.

Company	Individual	Shapley	%	EPM	%
Company 1	3,640	3,445	5.4	3,381	7.1
Company 2	14,684	13,350	9.1	13,284	9.5
Company 3	4,703	3,951	16.0	4,255	9.5
Company 4	2,043	1,835	10.2	1,848	9.5
Company 5	10,153	9,466	6.8	9,184	9.5
Company 6	4,826	4,380	9.2	4,365	9.5
Company 7	1,877	1,588	15.4	1,698	9.5
Company 8	0,332	0,301	9.3	0,300	9.5
Sum	**42,257**	**38,315**		**38,315**	
Stable		**Yes**		**Yes**	

Table 6: Distribution of costs according to Volume, Shapley and EPM and their savings as compared to the individual costs for instance I1 using backhauling routes.

Impact of several time periods

In the data covering the whole month, we have information about the time of the transportation. Hence we can analyze the behaviour of the cost allocations over four time periods, each covering one week. We compute cost allocations based on the Shapley value and the EPM. The results are shown in tables 7 (Shapley) and 8 (EPM). The Shapley value provides relatively stable results over the four weeks. One exception is that company 3 has a relatively large saving in week 1. For EPM we have that company 1 has lower savings in all weeks except week 2. Company 8 has smaller savings in three periods.

Instance/Company	I2	%	I3	%	I4	%	I5	%	Sum I2-I5	%
Company 1	0,828	4.5	0,975	3.3	0,958	5.1	0,858	6.4	3,619	4.8
Company 2	3,945	8.1	1,619	10.1	2,198	10.3	6,006	6.3	13,768	7.9
Company 3	0,769	20.1	1,023	8.1	1,321	9.9	1,045	13.5	4,159	12.5
Company 4	0,422	11.3	0,492	8.5	0,414	7.3	0,567	6.9	1,895	8.4
Company 5	2,270	4.9	2,418	4.5	2,711	5.4	2,439	6.9	9,838	5.4
Company 6	1,047	9.3	0,987	6.8	1,375	6.2	1,148	11.3	4,557	8.4
Company 7	0,532	12.5	0,358	14.5	0,372	17.9	0,350	14.8	1,613	14.7
Company 8	0,058	9.2	0,067	1.3	0,064	5.6	0,121	8.8	0,311	6.7
Sum	**9,872**		**7,940**		**9,412**		**12,534**		**39,758**	
Stable	**Yes**		**No**		**Yes**		**Yes**		**Yes**	

Table 7: Cost allocations for instances I2-I5 using Shapley.

Instance/Company	I2	%	I3	%	I4	%	I5	%	Sum I2-I5	%
Company 1	0,816	5.9	0,961	4.7	0,931	7.7	0,845	7.8	3,553	6.6
Company 2	3,899	9.2	1,667	7.4	2,254	8.0	5,908	7.8	13,728	8.2
Company 3	0,875	9.2	1,032	7.4	1,350	8.0	1,113	7.8	4,370	8.0
Company 4	0,432	9.2	0,498	7.4	0,411	8.0	0,561	7.8	1,902	8.0
Company 5	2,191	8.2	2,346	7.4	2,636	8.0	2,413	7.8	9,587	7.8
Company 6	1,049	9.2	0,981	7.4	1,348	8.0	1,192	7.8	4,571	8.1
Company 7	0,552	9.2	0,388	7.4	0,418	8.0	0,379	7.8	1,736	8.2
Company 8	0,058	9.2	0,067	1.2	0,064	6.0	0,123	7.8	0,312	6.4
Sum	**9,872**		**7,940**		**9,412**		**12,534**		**39,758**	
Stable	**Yes**		**Yes**		**Yes**		**Yes**		**Yes**	

Table 8: Cost allocations for instances I2-I5 using EPM.

In table 9, we compare the results for instances I2-I5 with instance I1 and I6. The values for Shapley in the average for instances I2-I5 are the same as for I6 which is due to the additivity property of Shapley. The structure of the allocations I1 and I6 are similar and the main difference is the increase in efficiency in I1 (39,253 kSEK instead of 39,758 kSEK), i.e. by viewing an aggregated problem. Using EPM in instances I1 and I6 clearly indicate that company 1 should be treated differently compared to the others.

Company	Shapley I1	%	EPM I1	%	Shapley aver I2-I5	%	EPM aver I2-I5	%	Shapley I6	%	EPM I6	%
Company 1	3,586	5.1	3,523	6.7	3,619	4.8	3,553	6.6	3,619	4.8	3,549	6.7
Company 2	13,528	8.5	13,549	8.4	13,768	7.9	13,728	8.2	13,768	7.9	13,751	8.0
Company 3	4,102	13.5	4,324	8.8	4,159	12.5	4,370	8.0	4,159	12.5	4,370	8.0
Company 4	1,889	8.6	1,884	8.8	1,895	8.4	1,902	8.0	1,895	8.4	1,902	8.0
Company 5	9,747	5.7	9,428	8.8	9,838	5.4	9,587	7.8	9,838	5.4	9,568	8.0
Company 6	4,503	9.2	4,522	8.8	4,557	8.4	4,571	8.1	4,557	8.4	4,573	8.0
Company 7	1,587	15.8	1,718	8.8	1,613	14.7	1,736	8.2	1,613	14.7	1,739	8.0
Company 8	0,310	6.9	0,304	8.8	0,311	6.7	0,312	6.4	0,311	6.7	0,306	8.0
Sum	**39,253**		**39,253**		**39,758**		**39,758**		**39,758**		**39,758**	
Stable	**Yes**		**Yes**		**Yes**		**Yes**		**Yes**		**Yes**	

Table 9: Cost allocations for instances I1, I6 and average of I2-I5 using Shapley and EPM.

Impact of geographical distribution

All the volumes contributed from each company do not improve the overall solution. In an attempt to test this aspect, we have removed all supplies/demands that do not directly change the solution. We have compared solutions in *opt1* and *opt3*, that is, the separate solutions for the individual companies with the collaborative solution in instance I1. Flows that have not changed from the individual solutions are removed. About 55% of the volumes are removed in this way. The remaining volumes are used to define instance I7.

In table 10, we provide the results from this instance. Column ”Fixed” represents the cost of the solution not changed against the individual solutions. Column ”Filtered” is the difference between columns ”Individual” and ”Fixed”. The value of total is the expected cost if each company worked independent. Columns ”Shapley” and ”EPM” give the cost allocation of the filtered problem. The total value under ”Shapley” is the coordinated value to be allocated to the participants. In columns ”Shapley+fixed” and ”EPM+fixed” we give the costs when the fixed value is added to the cost allocations. The columns ”%” and ”% (total)” give the relative and overall savings.

Using the Shapley value, the largest savings are for companies 3 and 8. However, the savings are all high in the range 14.0%-24.6%. The differences can be motivated by the location and importance of the companies. Using EPM, we can get the same savings for all companies, i.e., 17.8%. When we study the overall savings the spread is larger for both concepts. In fact, the structure is similar and the differences relatively small. If we compare the Shapley value for instance I7 (% (total)) and compare it with the Shapley value for instance I1, they are very similar. Using filtered data, we can first make a cost allocation for the filtered volumes providing a benefit. Then, the fixed part is added. This process does take the geographic location and its impact into account. It seems from the results, as if the Shapley value considers this aspect to a higher degree, than the EPM.

Company	Individual	Fixed	Filtered	Shapley	Shapley+fixed	%	% (total)	EPM	EPM+fixed	%	% (total)
Company 1	3,778	2,472	1,305	1,123	3,595	14.0	4.8	1,074	3,546	17.8	6.1
Company 2	14,859	7,072	7,788	6,410	13,482	17.7	9.3	6,405	13,477	17.8	9.3
Company 3	4,742	2,217	2,526	1,904	4,121	24.6	13.1	2,077	4,294	17.8	9.5
Company 4	2,067	0,824	1,243	1,064	1,888	14.4	8.6	1,022	1,846	17.8	10.7
Company 5	10,340	6,508	3,832	3,245	9,752	15.3	5.7	3,152	9,659	17.8	6.6
Company 6	4,959	2,290	2,670	2,210	4,499	17.2	9.3	2,196	4,485	17.8	9.6
Company 7	1,884	0,452	1,432	1,153	1,605	19.5	14.8	1,178	1,630	17.8	13.5
Company 8	0,333	0,233	0,100	0,077	0,310	22.8	6.9	0,082	0,315	17.8	5.3
Sum	**42,963**	**22,067**	**20,895**	**17,186**	**39,253**			**17,186**	**39,253**		
Stable				**Yes**				**Yes**			

Table 10: Cost allocations for instance I7.

Impact of coalition size

Using EPM, we have seen that company 1 has less cost savings than the other seven companies. In instance I8 we have removed company 1 and used cost allocations Shapley and EPM on this smaller problem, see table 11. Using EPM, we will get the same savings for the seven companies as given by EPM applied to instance I1. The reason is that the coalition of companies 2-8 provides the best value for this coalition and that they would like to break out. If a grand coalition is formed anyway, company 1 will get all of the increased improvement. However, it will not be enough to get the same improvements as the other seven companies. Using the Shapley value we find that the cost allocation is very similar to that but computed for instance I1.

Company	Individual	Shapley	%	EPM	%
Company 1					
Company 2	14,859	13,574	8.7	13,549	8.8
Company 3	4,742	4,092	13.7	4,324	8.8
Company 4	2,067	1,891	8.5	1,884	8.8
Company 5	10,340	9,742	5.8	9,428	8.8
Company 6	4,959	4,534	8.6	4,522	8.8
Company 7	1,884	1,587	15.8	1,718	8.8
Company 8	0,333	0,310	6.9	0,304	8.8
Sum	**39,185**	**35,730**		**35,730**	
Stable		**Yes**		**Yes**	

Table 11: Cost allocations for instance I8 using Shapley and EPM.

5 Practical aspects

There is great potential in collaborative planning. However, there are also a number of practical aspects to consider before such a process can be implemented. Below, we discuss a number of these aspects.

Business models

In a practical situation, the results in the previous section rely on the fact that all companies agree in advance to accept a cost allocation computed in any of the suggested approaches. In practice, this may be difficult as there is a large difference in size between companies or their position in the overall wood supply chain. It is likely that the largest companies are the key drivers and have a stronger position in a negotiation. They may work individually or in a group, depending on the business model used within the companies. Moreover, negotiations for wood bartering between companies is normally not generated by case studies such as in this paper. Instead, the initiative is taken by the timber managers representing each company. Personal relations are very important. The results from analyzes can be used in negotiations between several companies or by a freestanding organization for wood bartering. This would show the benefits and can be the decisive fact to initiate a collaboration. A company other than the forest companies may also be the key driver. It could be a transport organisation which takes the overall responsibility and does all the negotiating with each forest company.

Legality

One aspect of how the collaboration should be carried out concerns the law governing restrictive practices. Co-ordination of the transportation planning must be performed in a way that it cannot be interpreted as a formation of cartels. The exact ruling seems to be rather a grey zone, but the current interpretation by many companies is that collaboration and wood bartering between companies is allowed as long as it does not interfere with the overall wood market. For example, the companies are not allowed to collaborate in the buying of wood from forest land owners e.g. use the same price lists.

Tactical versus operational planning

In this paper, we have focused on collaboration between companies on a tactical level, and made a destination planning of the wood. This case study has involved truck transportation but can also be used when trucks are integrated with train and ship transportation. There are additional possibilities on a lower level of planning, where the actual routing of the trucks is carried out. This is often done by several hauliers and they get a certain part of the destinations. The hauliers may be working for a single or a combination of forest companies and they should of course be allocated destinations such that the potential for backhauling is maximized, in order to make the individual routing of logging trucks more efficient. This implies that the OR model [P2] described earlier should be used on a shorter planning horizon. The exchange of information can be made either between transport managers or between individual drivers.

Shared information and quality

A key factor for the successful co-ordination of transportation planning between companies is accurate and reliable information. In our case, study we made use of stored data in a common forest database. We could therefore easily compare our figures with transportation done in practice. However, for future planning, it is important that the supply and demand data is accurate. The demand is normally known with high accuracy. The supply is not known one month in advance and needs to be estimated. This must be based on the harvest plan, including locations and volumes of each assortment. If the planning horizon is shorter, actual harvested volumes can be used. Companies use different information systems and data collection and there is a need to set up a framework where all information is kept in a standardized way.

Valuation of wood

In our case study, the majority (70%) of the potential savings derive from the transportation of sawlogs. This is natural since the saw timber is bought and harvested with respect to a certain saw mill which is not always the closest one. This means, in this analysis, that those companies that prioritize buying sawlogs rather than pulplogs will have most benefits. It also means that it will be difficult to realize the entire potential. It is important to be able to make decisions on special bucking patterns, to produce e.g. certain dimensions for saw mills, once the destination of logs has been decided (as long as the assortment can be used). In Sweden, all companies use one standard to classify assortments. This contrasts with many other countries e.g. Finland where it is done within each company. A common classification is an important prerequisite for collaborative planning.

Decision support system

There are questions about what type of organization should collect sensitive information about supply, demand and costs and perform the collaborative planning. We believe that there needs to be an independent organisation to carry out and suggest wood bartering. This is already used in the operations to measure volume/weight of all delivered logs to mills and keeping track of all transportation carried out by trucks. This information is then used for invoicing between companies.

Companies may have different planning horizons or different storage strategies and this needs to be included in the OR model. In order for the planning to work, it is required that each company involved contributes with supply and demand such that it is in balance i.e. it is possible to establish a feasible plan for each participating company. The data that is provided depends on the agreement, but some companies may choose not to include parts of their supply and demand as they want to plan this elsewhere. There may be several reasons for this e.g. some very special assortments. This would of course limit the total potential.

In order for the collaborative planning to work, the planning and wood bartering needs to be analyzed continuously. Here, the actual transported volumes can be analyzed to compare against the underlying planning solutions. This could be done either by representatives from the co-operating companies, a company owned by the participating companies or by an external participant like a special logistic company.

6 Concluding remarks

It is very unusual that as many as eight forest companies together analyze the potential savings of cooperating within transportation planning. In practice, the number of companies is often limited to only two and they would agree only on bartering volumes. As more companies become part of the coordination it is not viable to agree only on volumes. The result of the analysis shows that there is a lot of money to save, up to 14.2% of the transportation cost. The transportation cost for these companies is about 60 million Euros and the potential saving relates to more than 8 million Euros. Additionally, the environmental effects of better cooperation between the companies are very positive with about a 20% reduction of emissions from the trucks.

We have studied a number of economic models and proposed a new cost allocation method, EPM, for how the costs can be distributed taking various properties of the planning problem into account. The concepts based on Shapley value, Nucleous, CGM and EPM provide stable cost allocations for the instances analyzed and are interesting as a basis for sharing costs or distributing savings. The cost allocations based on volume, ACAM and ECM give non-stable solutions for some instances. Moreover, volume and ECM give a large unmotivated spread in cost savings and are therefore less suitable.

We have analyzed the cost allocations based on the Shapley value and EPM on a set of instances. Coordination over several time periods (weeks) gives similar allocations as over one aggregated period (month). Including backhauling improves the efficiency but does not change the structure of cost allocations. The potential of cost savings depends on the geographical distribution of the companies in the coalition. By studying the supply and demand maps of the companies, it is relatively easy to understand the reason for spread in savings between the coalitions. We have proposed the EPM method because it makes it easier for the participants to agree, than by the use of any of the other considered methods. However, as companies become aware of the strategic importance of the geographical location, some companies may want a larger share of the cost savings. Then other cost allocation methods among those studied (or a hybrid) may be the preferred choice. The Shapley value seems to take this aspect into account. However, the Shapley value is more difficult to understand and harder to accept for planners. The two-step approach, which involves splitting the volumes into one fixed part and one part which represents the part that improves the planning, provides a powerful tool. In this way, it is easy to motivate planners that the cost allocation based on the non-fixed part of the volumes, should satisfy that all participants have the same relative savings, as achieved by EPM. Then, in the second step, the fixed part is added to form the total cost allocation. The cost allocations obtained when EPM is used in the first step are similar to the ones obtained when Shapley is used. However, EPM is easier to understand and more acceptable for the planners.

Situations when several companies are cooperating will become increasingly important in order to improve the transportation efficiency. Today, there exist systems and OR methodology that can establish coordinated plans. The work presented in this paper provide quantitative tools to make the cost sharing.

As future research, we will focus on four interesting aspects. First we intend to study how different business models will affect the overall system performance. Secondly, the question of how a negotiation process may take place when companies have different negotiating power, will be considered. Thirdly we want to study the effect of the geographical location of supply and demand points for a company and their impact on cost savings. Finally, we intend to develop a platform to be used in operational testing. This would include different planning periods, where we can test the sensitivity of information quality and the information provided by each company.

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